Synchronizing broadband chaotic systems to narrow-band signals

T. L. Carroll and G. A. Johnson
Code 6343, U.S. Naval Research Laboratory, Washington, D.C. 20375

(Received 9 July 1997)

We show that it is possible to take a signal from a chaotic drive system, pass it through a bandpass filter, and still use it to synchronize a chaotic response system. Narrow-band chaotic signals should be less sensitive to channel distortion and noise, so filtering should be useful for chaotic communications schemes. [S1063-651X(98)05002-8]

PACS number(s): 05.45.+b

The application of synchronized chaotic systems to communications has become a popular research topic [1–8]. Unfortunately, the broadband properties that make chaotic signals interesting also make the signals difficult to use in communications schemes. When real signals are transmitted through some medium (the channel), the properties of the medium usually vary with frequency. Broad-band signals, such as chaos, are greatly distorted by transmission through a real channel.

There has been some work on how to adapt chaotic receivers to correct for distortion due to transmission through a channel [9–12]. As an alternate approach, we ask how much information about the chaotic drive system is necessary in order to synchronize the receiver. One may be able to avoid some of the channel distortion problems by sending only this necessary information. We see in the work presented here that one may reduce the amount of information about the chaotic drive system by filtering the drive signal through a bandpass filter and still synchronize the response system. We do not need the full chaotic signal to synchronize the response system.

We have shown before that one may synchronize a chaotic response system to a signal that is a nonlinear function of signals in the chaotic drive system [13] or even a filtered version of a signal from the drive system [14,15] (when the chaotic systems were nonautonomous). Peng *et al.* [16] have shown a linear version of the result in Ref. [13]; one may synchronize a chaotic response system to a linear combination of signals from the drive system.

In this paper, we will look at drive-response systems of the form

$$\frac{d\mathbf{x}}{dt}=F(\mathbf{x}),\qquad \frac{d\mathbf{x}'}{dt}=F(\mathbf{x}')+\mathbf{b}(g_i-g'_i),$$
$$u=\mathbf{k}\cdot\mathbf{x},\qquad u'=\mathbf{k}\cdot\mathbf{x}', \tag{1}$$
$$\frac{d\mathbf{g}}{dt}=G(\mathbf{g},u),\qquad \frac{d\mathbf{g}'}{dt}=G(\mathbf{g}',u'),$$

where $\mathbf{x}$ is the drive system state vector, $\mathbf{x}'$ is the response system state vector, $\mathbf{k}$ and $\mathbf{b}$ are constant vectors, u is a scalar, G is a dynamical system, and g_i is a signal taken from the dynamical system G. We make a linear combination u of signals from the drive system $F(\mathbf{x})$ and drive the dynamical system G with u. We then take a signal from G, such as g_i, and transmit it to the response system. We set up the response system in the same way, then multiply the difference $(g_i'-g_i)$ by the vector $\mathbf{b}$ and add it to the response vector field. We find that if the response system (including G) has all Lyapunov exponents less than zero, the response will synchronize to the drive. In this work, we use bandpass filters for the dynamical system G, although other dynamical systems may work. A bandpass filter passes only a certain band of frequencies from the input signal.

In previous work [14,15], we synchronized circuits where the dynamical system $F(\mathbf{x})$ was a nonautonomous (periodically forced) system and the dynamical system G contained bandstop filters that removed the forcing frequency and the first several harmonics. The response system had its own periodic forcing source which replaced the part of the signal filtered out by G.

When we used filters with nonautonomous chaotic circuits, we were removing only a small amount of information about the chaotic drive system. We had a copy of the forcing function at our response system, so all that was lost was information on the phase of the forcing signal. In the present work, we will use bandpass filters with autonomous chaotic systems. We are now removing a large amount of information about the chaotic drive system. Causal filters (such as our bandpass filters) also change the dynamics of a signal [17]. There should be some minimum amount of information needed to synchronize a response system to a drive system.

As a numerical example, we link two Lorenz systems [18] through a bandpass filter. For our Lorenz example, the vector field F was given by $dx_I/dt=16(x_2-x_1)$, $dx_2/dt=-x_1x_3+45.92x_1-x_2$, and $dx_3/dt=x_1x_2-4x_3$. The scalar u was $u=k_1x_1+k_2x_2+k_3x_3$. The filter G was described by

$$\frac{dg_1}{dt}=-\frac{2}{R_1C}\,g_1-\left(\frac{1}{2R_2C}\right)\left(\frac{1}{R_3C}-\frac{1}{R_1C}\right)g_2-\left(\frac{1}{R_1C}\right)\frac{du}{dt},$$
$$\frac{dg_2}{dt}=g_1. \tag{2}$$

At the response system, we took the difference $(g_2'-g_2)$, multiplied by a vector $\mathbf{b}=(b_1,b_2,b_3)$ and added the result to the response vector field.

Equation (2) represents a second-order bandpass filter [19]. The center frequency f_c is passed with a gain of 1, while other frequencies are attenuated by an amount that increases as they become farther from f_c. The constants

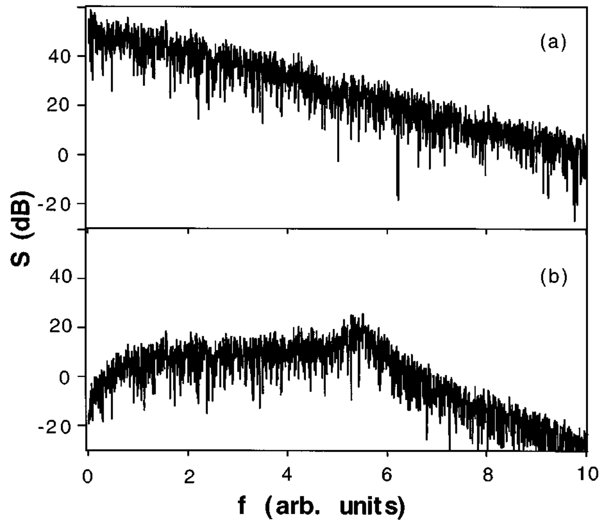

FIG. 1. (a) Power spectrum S for the unfiltered signal u from the numerical Lorenz system. (b) Power spectrum S for the filtered signal g_2 from the numerical Lorenz system.

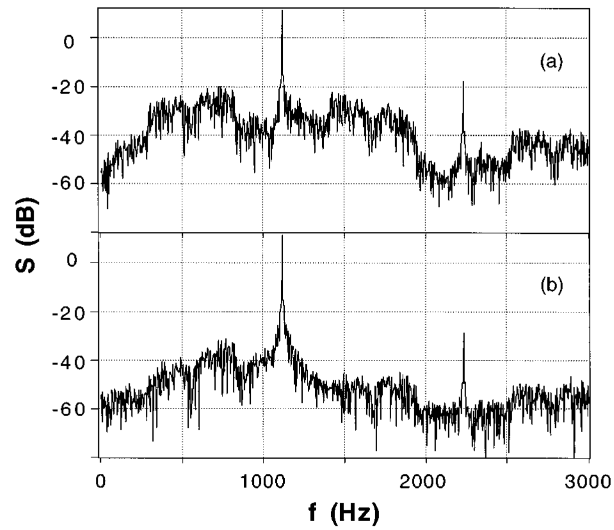

FIG. 3. (a) Power spectrum S for the unfiltered signal u from the piecewise linear Rössler circuit. (b) Power spectrum S for the filtered signal g_f from the piecewise linear Rössler circuit. Frequencies outside the pass band have been reduced by 15–20 dB (a factor of 30–100) in power

were $C=1$, $R_1=3.183$, and $R_2=6.366$. R_3 was used to vary the center frequency of the filter, so for a center frequency of f_c, $R_3=R_1/[-1+4(\pi f_c)^2 R_1 R_2]$. For these parameters, the Q factor of the filter was 20 (the Q factor is the center frequency f_c divided by the bandwidth). The center frequency f_c of the filter was varied between 0.1 and 10. Numerical integration was carried out by a fourth-order Runge-Kutta integration routine [20]. The component of **k** and **b** [16] were chosen by a numerical minimization routine to make the largest Lyapunov exponent for the response system less than zero.

Figure 1(a) shows the power spectrum of the signal u for the Lorenz system. Figure 1(b) shows the power spectrum of the signal g_2 (the filtered version of u), for a center frequency $f_c=5.44$. The components of **k** and **b** were $k_1=273.0212$, $k_2=23.265\,57$, $k_3=16.247\,05$, $b_1=18.936\,43$, $b_2=20.519\,21$, and $b_3=-3.043\,97$. For these parameters, the largest Lyapunov exponent for the response system was -4.95. Figure 2 shows the synchronization of the response system to the drive system. We found that with the above parameters, the response system was stable for values of f_c, ranging from about 1 to about 9.

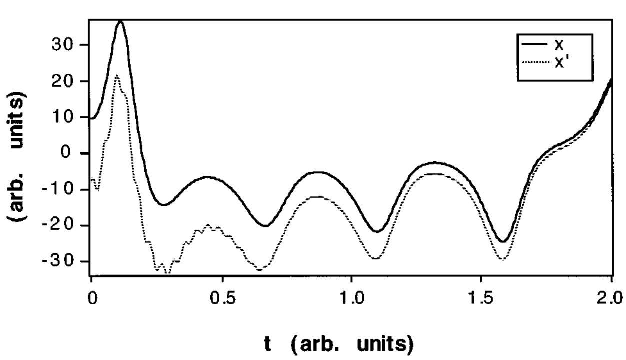

FIG. 2. The solid line shows the drive Lorenz system, while the dotted line shows the response Lorenz system synchronizing to the drive.

We also built a set of electronic circuits that could be synchronized through a filter. Our drive and response circuits were piecewise linear circuits [21] whose attractors resembled the Rössler attractor. We filtered out all but the central peak in the transmitted signal spectrum [Fig. 3(a)] by bandstop filtering the signal u from the Rössler drive circuit and subtracting from the unfiltered signal. We found that this arrangement was more stable for our circuits than a band-pass filter.

Our drive circuit vector field was described by $dx_1/dt=-\gamma(0.05x_1+0.5x_2+x_3)$, $dx_2/dt=-\gamma(-x_1-0.11x_2)$, and $dx_3/dt=-\gamma[x_3+h(x_1)]$, where $h(x)=0$ if $x\leqslant 3$ and $h(x)=15(x-3)$ if $x>3$. The time factor γ was $10^4\ \mathrm{s}^{-1}$.

The filter G was described by

$$\frac{dg_1}{dt}=-\left(\frac{1}{RC}\right)\left(\frac{3g_1}{1+\alpha}+g_2+\frac{\beta}{1+\alpha}u\right)-\frac{\beta}{1+\alpha}RC\frac{d^2u}{dt^2},$$

$$\frac{dg_2}{dt}=\frac{1}{RC}g_1,$$

$$g_f=u+g_2, \tag{3}$$

where the narrow-band output signal was g_f. The filter Q was given by $(\alpha+1)/3$, and the filter gain was $-\beta/(1+\alpha)$. The Q factor was set to $7(\alpha=20)$ and the gain to $-1(\beta=1+\alpha)$. The filter center frequency f_c (the frequency at which the bandstop output was zero) was $1/(2\pi RC)$. We set the center frequency to coincide with the main frequency peak in the spectrum of the signal u from the circuit, at 1145 Hz. Figure 3(a) shows the unfiltered power spectrum of u, while Fig. 3(b) is the power spectrum of the filtered signal g_f. We transmitted the signal g_f to the response circuit.

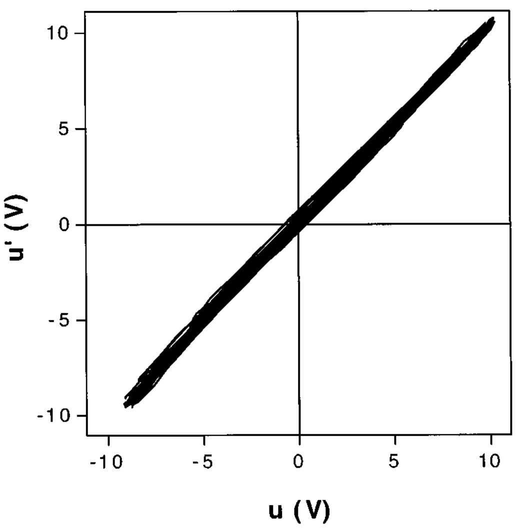

FIG. 4. Unfiltered signal u' from the response Rössler circuit vs unfiltered signal u from the drive Rössler circuit.

The response circuit was piecewise linear, so we were able to estimate the stability of the response circuit by finding a Jacobian for the case $g(x)=0$. This Jacobian was constant, so we used the largest real part of the eigenvalues of the Jacobian as an estimate of the stability of the response circuit. We varied the components of **k** and **b** to find a stable response system.

The response circuit was stable for $k_1=-1.9$, $k_2=1.1$, $k_3=1$, $b_1=1$, $b_2=1$, and $b_3=1$. The largest real part of the eigenvalues for the response circuit was -1170. Figure 4 shows v from the response circuit vs u from the drive circuit, showing synchronization.

One might ask why a narrow-band filter passes enough information to synchronize a response circuit. We may divide the chaotic motion into motion on a synchronization manifold (where the systems are synchronized) and motion transverse to the synchronization manifold. Hunt and Ott [22,23] have stated that one gets the optimal average of any smooth function of a system state by averaging over a low-period orbit, so if the averages over several low-period orbits of the Lyapunov exponents transverse to the synchronization manifold are negative, we should see synchronization [24,25]. For the piecewise linear Rössler circuit, all of the low-period unstable orbits have a large spectral component at the main peak in the Rössler spectrum, so if we filter at this peak frequency, we can stabilize all of the low-period orbits at once.

The periodic orbits for the Lorenz system contain many different frequencies. Although the global Lyapunov exponents for the Lorenz response system above are always negative, the local Lyapunov exponents are sometimes negative and sometimes positive. We are able to make the average Lyapunov exponent negative because we are stabilizing one or more low-period orbits, which dominate the average. We added 1% random noise to the Lorenz simulation and saw no evidence of bursting away from synchronization [24,25].

Using a narrow-band signal to synchronize broadband systems has some obvious advantages for applications in communications. Reduced bandwidth means that the transmitted signal will suffer less distortion. Filtering the transmitted signal at the receiver will remove much of the noise picked up in transmission. One could even synchronize multiple response systems to the same chaotic signal filtered at different frequencies. By comparing the different response systems, one might be able to reduce the effects of frequency-dependent noise.

Adding filters to synchronized chaotic systems does bring some loss of stability, so the filtered systems will take longer to synchronize and will be less robust to noise that is not filtered out. One may understand this loss of stability by considering the filtering as a convolution of a time series with some filter function. The narrower the pass band of the filter, the longer the time will be in which the filter averages the incoming signal. Long-time averages mean that the filter cannot respond quickly to changes in the incoming signal, so the response system is less stable. Rulkov [12] has described an alternate method that avoids this stability problem by designing narrow-band chaotic systems.

[1] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[2] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).
[3] T. L. Carroll and L. M. Pecora, Physica D **67**, 126 (1993).
[4] L. Kocarev and U. Parlitz, Phys. Rev. Lett. **74**, 5028 (1995).
[5] U. Parlitz, L. Kocarev, T. Stojanovski, and H. Prekel, Phys. Rev. E **53**, 4351 (1996).
[6] K. Pyragas, Phys. Lett. A **181**, 203 (1993).
[7] R. H. Sherman and J. Gullicksen, Proc. SPIE **2038**, 141 (1993).
[8] L. O. Chua, L. Kocarev, K. Eckart, and M. Itoh, Int. J. Bifurcation Chaos Appl. Sci. Eng. **2**, 705 (1992).
[9] A. V. Oppenheim, K. M. Cuomo, R. J. Barron, and A. E. Freedman, *Proceedings of the Third Technical Conference on Nonlinear Dynamics and Full Spectrum Processing*, AIP Conf. Proc. No. 296, edited by R. A. Katz (AIP, New York, 1995).
[10] L. O. Chua, T. Yang, G.-Q. Zhong, and C. W. Wu, IEEE Trans. Circuits Syst. **43**, 862 (1996).
[11] T. L. Carroll, Phys. Rev. E **53**, 3117 (1996).
[12] N. F. Rulkov (unpublished).
[13] T. L. Carroll, J. F. Heagy, and L. M. Pecora, Phys. Rev. E **54**, 4676 (1996).
[14] T. L. Carroll, Phys. Rev. E **50**, 2580 (1994).
[15] T. L. Carroll, IEEE Trans. Circuits Syst. **42**, 105 (1995).
[16] J. H. Peng, E. J. Ding, M. Ding, and W. Yang, Phys. Rev. Lett. **76**, 904 (1996).
[17] R. Badii, G. Broggi, B. Derighetti, M. Ravini, S. Ciliberto, and A. Politi, Phys. Rev. Lett. **60**, 979 (1988).
[18] E. N. Lorenz, J. Atmos. Sci. **20**, 130 (1963).
[19] U. Tietze and C. Shenk, *Electronic Circuits* (Springer, Berlin, 1991).

[20] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes* (Cambridge University Press, New York, 1990).
[21] T. L. Carroll, Am. J. Phys. **63**, 377 (1995).
[22] B. R. Hunt and E. Ott, Phys. Rev. Lett. **76**, 2254 (1996).
[23] B. R. Hunt and E. Ott, Phys. Rev. E **54**, 328 (1996).
[24] P. Ashwin, J. Buescu, and I. Stewart, Phys. Lett. A **193**, 126 (1994).
[25] J. F. Heagy, T. L. Carroll, and L. M. Pecora, Phys. Rev. E **52**, 1253 (1995).